VARIATIONAL AND EXTREMUM PRINCIPLES IN MACROSCOPIC SYSTEMS

Edited by

Stanislaw Sieniutycz
Faculty of Chemical & Process Engineering,
Warsaw University of Technology,
Warsaw, Poland

Henrik Farkas
Institute of Physics,
Budapest University of Technology and Economics,
Budapest, Hungary

2005

ELSEVIER

AMSTERDAM – BOSTON – HEIDELBERG – LONDON – NEW YORK – OXFORD
PARIS – SAN DIEGO – SAN FRANCISCO – SINGAPORE – SYDNEY – TOKYO

FOREWORD

The concept of a variational principle as a fundamental characteristic of all phenomena is remarkable, if one steps back and considers what conceptual basis it implies. At the first encounter, it may seem obvious and almost trivial: of course, every process is an extremum of *something*, because we think we can find a suitable "something", a set of constraints, that makes the answer come out right. That is, we are inclined to think that we could find constraints—and a suitable variational function or functional—for any given problem that would assure that the process satisfied a variational principle. The not-so-subtle difficulty, in many situations easy to recognize, but usually not at all easy to resolve, is determining what the relevant constraints are.

This line of reasoning leads to a question that underlies many of the chapters in this volume. Is it true that every process can be described by some pathway that makes the entropy change an extremum? That is, of course, not quite how the question has to be expressed. Rather, we should ask this: *Is it true that all processes can be described by a pathway that makes the entropy change an extremum, subject to a specific set of constraints*? Is there a variational principle for all possible processes, if we can determine the constraints suitably? And if so, what quantities can be the basic ones for such a principle? We have principles of least action, of least time and of least (or most) entropy production. Are there relations among these that are stronger than mere analogies? Are there other quantities for which there are variational principles?

But let us turn to our original statement: the concept, valid in a remarkable range of situations, that we can find and determine *some* quantitative property that is an extremum for many, possibly all processes we can describe, is indeed amazing. We are so familiar with this idea that we take it very much as a natural, given property of the physical world, even extending it as is done here to ecosystems and economic behavior. Using it as we do, we may appreciate the beauty of any variational principle, yet, even if we appreciate its beauty, that very familiarity hides from us the amazement we should have at our capacity to discover it at all. We should realize how remarkable it is that the human mind could recognize and then generalize the concept. Perhaps the easiest to discover was the principle of least time; this minimizes something in our direct experience, even though it is not so obvious that we might conceive of time trajectories different from the ones we observe. But variational principles for action and for entropy take us beyond direct experience. We have no sensors in our bodies for action, no organ that responds to entropy. These are abstract creations of human imagination that we can quantify. Then to go beyond that stage and discover that they are the bases for variational principles is a thing for us to see with wonder.

One further observation: most of the chapters in this volume deal with *descriptive* variational methods, some with methodology and some with applications. A few of the chapters follow the other kind of use of variational methods, what we may call the *prescriptive*. This direction is perhaps epitomized by optimal control theory. Here we use the power of variational calculus to tell us what *we* should do in order to achieve some goal. Here, we can sometimes open new directions by recognizing that some variable previously considered beyond our control could, in fact, become a control variable and enable us to construct entirely new kinds of devices. Perhaps, in using this volume, some readers will see ways innovating by pursuing directions such as that, beginning with what they learn from the rich material presented here.

R. Stephen Berry
The University of Chicago